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Anomaly in Neutrino–Nucleon Scattering and Vectorlike Families

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Abstract

The Extended Supersymmetric Standard Model (ESSM), motivated on several grounds, introduces two vector-like families [$\mathbf{16} + \overline{\mathbf{16}}$ of SO(10)] with masses of order one TeV. Following earlier work, a successful pattern for fermion masses and mixings is proposed within a unified SO(10)-framework incorporating ESSM, which makes eight predictions, in good accord with observations, including $V_{cb} \approx 0.036$, and $\sin^2 2\theta_{\nu_\mu \nu_\tau} \approx 1$. It is noted that the anomaly in ν_μ–nucleon scattering, reported recently by the NuTeV experiment, can be understood simply within the ESSM/SO(10)-framework and the pattern of the fermion mass-matrices proposed here, in terms of a reduction of the $Z^0 \to \nu_\mu \overline{\nu}_\mu$ coupling (leaving $\sin^2\theta_W$ and $g_L^{\rm eff}$ unaltered). This explanation leads us also to predict (a) a correlated *reduction* in LEP neutrino counting from $N_\nu = 3$ (which is in good agreement with the data), and (b) small departures in lepton universality in charged current processes. These and the searches for the vectorlike families at the LHC and the NLC can clearly test our model.

1 Introduction

The recently reported NuTeV result on ν_μ-nucleon scattering [1] suggests that quite possibly there is an anomaly in $(\sigma_{NC}/\sigma_{CC})$-ratios ($R_\nu$ and $R_{\bar{\nu}}$) compared to expectations of the Standard Model. If the result persists against (even) more precise data, and improved theoretical scrutiny, it would clearly have some profound implications. We plan to discuss one of these in the context of an idea proposed some time ago.

The results on R_ν and $R_{\bar{\nu}}$ have been interpreted in Ref. [1] to reflect either (a) a higher on-shell value of $\sin^2\theta_W$ which is at 3σ above the prediction of the Standard Model (SM), or (b) a reduced coupling of the left-handed quarks to $Z^0(g_L^{\rm eff})$, compared to the SM value for the same. A third possibility has also been mentioned in the context of a two-parameter fit corresponding to a reduced overall strength (ρ_0) of the neutral current four fermion coupling together with a possible non-standard value of $\sin^2\theta_W$. The purpose of this note is to point out that the NuTeV anomaly, interpreted solely as a reduction in the overall strength of the $Z^0 \to \nu_\mu\bar{\nu}_\mu$ coupling (leaving $\sin^2\theta_W$ and $g_L^{\rm eff}$ unaltered) can be understood simply in terms of an old idea, that is motivated on several grounds (see below) [2, 3]. This is the so-called "Extended SuperSymmetric Standard Model" (ESSM), which introduces two complete vectorlike families of quarks and leptons – denoted by $Q_{L,R} = (U, D, N, E)_{L,R}$ and $Q'_{L,R} = (U', D', N', E')_{L,R}$ – with masses of order few hundred GeV to one TeV. Both Q_L *and* Q_R transform as (2,1,4), while Q'_L *and* Q'_R transform as (1,2,4) of the quark-lepton unifying symmetry G(224)=SU(2)$_L\times$SU(2)$_R\times$SU(4)^C. Thus, together, they transform as a pair $\mathbf{16}+\overline{\mathbf{16}}$ of SO(10), to be denoted by $\mathbf{16_V} = (Q_L|\bar{Q}'_R)$ and $\overline{\mathbf{16}}_\mathbf{V} = (\bar{Q}_R|Q'_L)$. The subscript "V" signifies two features: (a) $\overline{\mathbf{16}}_\mathbf{V}$ combines primarily with $\mathbf{16_V}$, so that the pair gets an SO(10)-invariant (thus SU(2)$_L\times$U(1)-invariant) mass-term of the form $M_V\mathbf{16_V}\cdot\overline{\mathbf{16}}_\mathbf{V}+h.c. = M_V(\bar{Q}_RQ_L + \bar{Q}'_RQ'_L) + h.c.$, at the GUT scale, utilizing for example the VEV of an SO(10)-singlet, where $M_V \sim$ few hundred GeV to one TeV [4], (b) since Q_L and Q_R are doublets of SU(2)$_L$, the massive four-component object $(Q_L \oplus Q_R)$ couples *vectorially* to W_L's; likewise $(Q'_L \oplus Q'_R)$ couples *vectorially* to W_R's. Hence the name "vectorlike" families.

It has been observed in earlier works [5] that addition of complete *vectorlike* families [$\mathbf{16}+\overline{\mathbf{16}}$ of SO(10)], with masses $\gtrsim 200$ GeV to one TeV (say), to the Standard Model naturally satisfies all the phenomenological constraints so far. These include: (a) neutrino-counting at LEP [6] (because $M_{N,N'} > m_Z/2$), (b) measurement of the ρ-parameter [because the SO(10)-invariant mass for the vectorlike families ensure up-down degeneracy – i.e., $M_U = M_D$, etc. – to a good accuracy], and (c) those of the oblique electroweak parameters [7] (for the same reasons as indicated above) [6]. We will comment in just a moment on the theoretical motivations for ESSM. First let us note why ESSM is expected to be relevant to the NuTeV anomaly and why it would simultaneously have implications for the LEP neutrino-counting. As a central feature, ESSM assumes that the three chiral families (e, μ and τ) receive their masses primarily (barring corrections $\lesssim$ a few MeV) through their mixings with the two vectorlike families [2, 3]. As we will explain in Sec. 2, this feature has the advantage that it automatically renders the electron family massless (barring corrections as mentioned above); and at the same time it naturally assigns a large hierarchy between the muon and the tau family masses, *without* putting in such a hierarchy in the respective Yukawa couplings [2, 3, 8]. In short, ESSM provides a simple reason for the otherwise mysterious interfamily mass hierarchy, i.e., ($m_{u,d,e} \ll m_{c,s,\mu} \ll m_{t,b,\tau}$). Now, since the chiral families get masses by mixing with the vectorlike families, the observed neutrinos ν_i naturally mix with the heavy neutrinos N_L and N'_L belonging to the families Q_L and Q'_L, respectively. The mixing parameters get determined in terms of fermion masses and mixings. As we will explain, it is the mixing of ν_μ and likewise of ν_τ with the SU(2)$_L$-*singlet* heavy lepton N' belonging to the family Q'_L, that reduces the overall strengths of the couplings (i) $Z^0 \to \nu_\mu \bar{\nu}_\mu$, (ii) $Z^0 \to \nu_\tau \bar{\nu}_\tau$, as well as of (iii) $W^+ \to \mu^+ \nu_\mu$, and (iv) $W^+ \to \tau^+ \nu_\tau$, compared to those of the Standard Model, *all in a predictably correlated manner.* The forms of the couplings remain, however, the same as in the Standard Model.

In accord with the interfamily hierarchy of fermion masses and mixings, the reduction in the couplings as above is found to be family-dependent, being maximum in the ν_τ, intermediate in ν_μ and negligible (< one part in a million) in the ν_e-channel.

These effects would manifest themselves as (a) a deficit in the LEP neutrino-counting from the Standard Model value of $N_\nu \equiv N_{\nu_e} + N_{\nu_\mu} + N_{\nu_\tau} = 3$, (b) as a correlated reduction in the strength of $\nu_\mu N \to \nu X$ interaction (which is relevant to the NuTeV anomaly), and also as (c) departures from universality in the tau and muon lifetimes as well as in $\pi \to l\nu$-decays. Qualitative aspects of these effects arising from ν_i-N'-mixing (without a quantitative hold on the reduction in the $\nu_\mu \bar{\nu}_\mu$ and $\nu_\tau \bar{\nu}_\tau$–couplings to Z^0) were in fact noted in an earlier work [5] almost ten years ago. In that work, motivated by an (overly) simplified version of understanding the inter-family hierarchy, the effect of the $\nu_\mu \bar{\nu}_\mu$-channel was considered to be too small. Two interesting developments have, however, taken place in the meanwhile. First, SuperK discovered atmospheric neutrino oscillations, showing that ν_μ oscillates very likely into ν_τ with a surprisingly large oscillation angle: $\sin^2 2\theta^{\rm osc}_{\nu_\mu \nu_\tau} \gtrsim 0.92$ [9]. Second, motivated in part by the SuperK result, an economical SO(10)-framework has been proposed in the context of a minimal Higgs system ($\mathbf{10_H}$, $\mathbf{16_H}$, $\overline{\mathbf{16}}_\mathbf{H}$ and $\mathbf{45_H}$) to address the problem of fermion masses and mixings [10]. Within this framework, a few variant patterns of fermion mass-matrices are possible, each of which is extremely successful in describing the masses and mixings of all fermions including neutrinos. For example, the pattern exhibited in [10] makes eight predictions, including $V_{cb} \approx 0.042$ and $\sin^2 2\theta^{\rm osc}_{\nu_\mu \nu_\tau} \approx 0.85$-$0.99$, all in accord with the data to within 10% [11]. Interestingly, it turns out that the variant patterns of fermion mass-matrices, cast within the ESSM framework, can in fact be distinguished by NuTeV-type experiments. In particular, we show, that in the context of a close variant of [10], extended to ESSM, which preserves the successes of [10], the *ratio* of ν_μ-N' and ν_τ-N' mixings and also the μ-τ mixing are surprisingly large. Because of this, it turns out that one can account for the NuTeV anomaly, and simultaneously predict a deficit in the LEP neutrino-counting, *and* also departures from universality in the tau and muon lifetimes, as well as in $\pi \to l\nu$-decays. All of these are presently in reasonable accord with the data, but can be checked with further improvements.

Before discussing the relevance of ESSM to the NuTeV anomaly, a few words about motivations for ESSM might be in order. Note that it, of course, preserves all the merits of

MSSM as regards gauge coupling unification and protection of the Higgs masses against large quantum corrections. Theoretical motivations for the case of ESSM arise on several grounds: (a) It provides a better chance for stabilizing the dilaton by having a semi-perturbative value for $\alpha_{\rm unif} \approx 0.25$ to 0.3 [3], in contrast to a very weak value of 0.04 for MSSM; (b) It raises the unification scale M_X [3, 12] compared to that for MSSM and thereby reduces substantially the mismatch between MSSM and string unification scales [13]; (c) It lowers the GUT-prediction for $\alpha_3(m_Z)$ compared to that for MSSM [3], as needed by the data; (d) Because of (b) and (c), it naturally enhances the GUT-prediction for proton lifetime compared to that for MSSM embedded in a GUT [10, 14], also as needed by the data [15]; and finally, (e) as mentioned above, it provides a simple reason for interfamily mass-hierarchy. In this sense, ESSM, though less economical than MSSM, offers some distinct advantages. The main point of this paper is to note that it can also provide a simple explanation of the NuTeV-anomaly. It, of course, offers a clear potential for the discovery of a host of vectorlike quarks and leptons at the LHC and possibly the NLC. In an accompanying paper [16], we have noted how ESSM can account for the indicated anomaly in muon $(g-2)$ [17] and how it can be probed efficiently through improvements in forthcoming measurements of $(g-2)_\mu$ as well as searches for $\tau \to \mu\gamma$ and $\mu \to e\gamma$.

2 Fermion Masses and Mixings in ESSM

Following the discussion in the introduction (see Ref. [3] for details and notation), the 5×5 mass-matrix involving the three chiral ($q^i_{L,R}$) and the two vectorlike families ($Q_{L,R}$ and $Q'_{L,R}$) is assumed to have the see-saw form:

$$M^{(0)}_{f,c} = \begin{array}{c} \\ \bar{q}^i_R \\ \bar{Q}_R \\ \bar{Q}'_R \end{array} \begin{array}{c} \begin{array}{ccc} q^i_L & Q_L & Q'_L \end{array} \\ \left(\begin{array}{ccc} 0_{3\times 3} & X_f\langle H_f\rangle & Y_c\langle H_s\rangle \\ Y'^{\dagger}_c\langle H_s\rangle & z_c\langle H_V\rangle & 0 \\ X'^{\dagger}_f\langle H_f\rangle & 0 & z'_f\langle H_V\rangle \end{array}\right) \end{array} . \qquad (1)$$

Here the symbols q, Q and Q' stand for quarks as well as leptons; i=1, 2, 3 corresponds to the three chiral families. The subscript f for the Yukawa-coupling column matrices X_f and X'_f denotes u, d, l or ν, while $c = q$ or l denotes quark or lepton color. The fields H_f with $f = u$ or d denote the familiar two Higgs doublets, while H_s and H_V are Higgs Standard Model singlets [18], whose VEVs are as follows: $\langle H_V\rangle \equiv v_0 \sim 1$ TeV, $\langle H_s\rangle \equiv v_s \gtrsim \langle H_u\rangle \equiv v_u \sim 200$ GeV $\gg \langle H_d\rangle \equiv v_d$. The zeros in Eq. (1), especially the direct coupling terms appearing in the upper 3×3 block, are expected to be corrected so as to leads to masses $\lesssim$ a few MeV.

The parametrization in Eq. (1) anticipates that differences between z_c and z'_f, between X_f and X'_f, and between Y_c and Y'_c may arise at the electroweak scale in part because renormalization effects distinguish between $Q_{L,R}$, which are SU(2)$_L$-doublets, and $Q'_{L,R}$, which are SU(2)$_L$-singlets (see Eq. (10) of Ref. [3]), and in part because (B-L)-dependent and L-R as well as family-antisymmetric contributions may arise effectively by utilizing the VEV of a **45**-plet, sometimes in conjunction with that of a $\mathbf{10_H}$ (see Refs [10] and [16] for details), which can introduce differences between X_f and X'_f, etc.

Denoting $X_f^T = (x_1, x_2, x_3)_f$ and $Y_c^T = (y_1, y_2, y_3)_c$, it is easy to see [2,3,8] that regardless of the values of these Yukawa couplings, one can always transform the basis vectors $\bar{q}_R^i$ and q_L^i so that Y_c^T transforms into $\hat{Y}_c^T = (0,0,1)y_c$, X_f^T simultaneously into $\hat{X}_f^T = (0, p_f, 1)x_f$, $X_f'^\dagger$ into $\hat{X}_f'^\dagger = (0, p'_f, 1)x'_f$ and $Y_c'^\dagger$ into $\hat{Y}_c'^\dagger = (0,0,1)y'_c$. It is thus apparent why one family remains massless (barring corrections of $\lesssim$ a few MeV), despite lack of any hierarchy in the Yukawa couplings $(x_i)_f$ and $(y_i)_c$, etc. This one is naturally identified with the electron family. To a good approximation, one also obtains the relations [2,3]: $m^0_{c,s,\mu} \approx m^0_{t,b,\tau}(p_f p'_f/4)$. Even if p_f, p'_f are not so small (e.g. suppose $p_f, p'_f \sim 1/2$ to $1/4$), their product divided by four can still be pretty small. One can thus naturally get a large hierarchy between the masses of the muon and the tau families as well.

As shown in Ref. [16], the SO(10) group-structure of the (2,3)-sector of the effective 3×3 mass matrix for the three chiral families, proposed in Ref. [10], can be preserved (to a good approximation) for the case of ESSM, simply by imposing an SO(10)-structure on the off-diagonal Yukawa couplings of Eq. (1), that is analogous to that of Ref. [10] (see [19]), while small entries involving the first family can be inserted, as in Ref. [10], through higher dimensional operators. (We refer the reader to Ref. [16] and to a forthcoming paper [20] devoted entirely to "fermion masses in ESSM" for more details.)

It is the Dirac mass-matrices of the neutrinos and of the charged leptons that are relevant to the present paper. In the hat-basis mentioned above, where the first family is (almost) decouples from the two vectorlike families, the Dirac mass-matrix of the neutrinos (following notations of Ref. [10] and [16]) is given by [21]:

$$M_\nu^D = \begin{array}{c} \\ \bar{\nu}_R^e \\ \bar{\nu}_R^\mu \\ \bar{\nu}_R^\tau \\ \bar{N}_R \\ \bar{N}'_R \end{array} \begin{array}{c} \begin{array}{ccccc} \nu_L^e & \nu_L^\mu & \nu_L^\tau & N_L & N'_L \end{array} \\ \left(\begin{array}{ccc} \multicolumn{3}{c}{0_{3\times3}} & \begin{pmatrix} 0 \\ p_\nu \\ 1 \end{pmatrix}\kappa_u^\nu & \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\kappa_s^l \\ (0 \quad 0 \quad 1)\,\kappa_s'^l & M_N & 0 \\ (0 \quad p'_\nu \quad 1)\,\kappa_u'^\nu & 0 & M_{N'} \end{array} \right) \end{array}. \tag{2}$$

Here, $\kappa_u^\nu \equiv x_\nu \langle H_u \rangle$, $\kappa_u'^\nu \equiv x'_\nu \langle H_u \rangle$, $\kappa_s^l \equiv y_l \langle H_s \rangle$, $\kappa_s'^\nu \equiv y'_l \langle H_s \rangle$, $M_N \approx M_E = z_l \langle H_V \rangle$, $M_{N'} \approx M_{E'} = z'_l \langle H_V \rangle$. The mass matrix for the charged leptons is obtained by replacing the suffix ν by l and u by d, so that $H_u \to H_d$, $\kappa_u^\nu \to \kappa_d^l$, but $\kappa_s^l \to \kappa_s^l$, etc. Analogous substitution give the mass matrices for the up and down quarks. We stress that *the parameters of the mass-matrices of the four sectors u, d, l and ν, and also those entering into X versus X' or Y versus Y' in a given sector, are of course not all independent,* because a large number of them are related to each other at the GUT-scale by the group theory of SO(10) and the representation(s) of the relevant Higgs multiplets [22]. For convenience of writing, we drop the superscript ν on kappas, from now on.

We now proceed to determine some of these parameters in the context of a promising SO(10)–model, which would turn out to be especially relevant to the NuTeV–anomaly and the LEP neutrino counting.

3 Determining the Parameters Relevant to NuTeV Within a Predictive SO(10) Framework Based on ESSM

Following the approach of [10] and keeping in mind the NuTeV anomaly, we now present a concrete example wherein the effective mass-matrices for ESSM, exhibited in Eq. (1) and (2), emerge from a unified SO(10) framework. The pattern of the mass-matrices for the three light families in the (u, d, l, ν)-sectors, which result from this example upon integrating out the heavy families (Q and Q'), turns out to be a *simple variant* of the corresponding pattern presented in Ref. [10]. The variant preserves the economy (in parameters) and the successes of Ref. [10] as regards predictions of the masses and mixings of quarks as well as leptons including neutrinos; these include $V_{cb} \simeq 0.04$ and $\sin^2 2\theta^{\mathrm{osc}}_{\nu_\mu\nu_\tau} \approx 1$. At the same time, the variant turns out to be relevant (quantitatively) to account for the NuTeV-anomaly and simultaneously for the LEP data on neutrino counting.

Let us assume for the sake of simplicity that the electron family is (almost) decoupled from the heavy families (Q and Q') in the gauge-basis – that is to say, the gauge and the hat–basis (defined earlier) are essentially the same, so that Eq. (2) holds to an excellent approximation, already in the gauge basis. Consider then the following superpotential (see Ref. [16] for details), which involve the μ and the τ families ($\mathbf{16}_2$ and $\mathbf{16}_3$) and the two vector-like families ($\mathbf{16}_V$ and $\overline{\mathbf{16}}_V$):

$$\begin{aligned} W_{\mathrm{Yuk}} &= h_V \mathbf{16}_V \overline{\mathbf{16}}_V H_V + h_{3V} \mathbf{16}_3 \mathbf{16}_V \mathbf{10}_H + h'_{3V} \mathbf{16}_3 \mathbf{16}_V \mathbf{16}_H \mathbf{16}^d_H / M \\ &+ h_{3\bar{V}} \mathbf{16}_3 \overline{\mathbf{16}}_V H_s + h_{2V} \mathbf{16}_2 \mathbf{16}_V \mathbf{10}_H \mathbf{45}_H / M + h'_{2V} \mathbf{16}_2 \mathbf{16}_V \mathbf{16}^d_H \mathbf{16}_H / M \ . \end{aligned} \tag{3}$$

Here, $\langle \mathbf{16}_H \rangle \sim \langle \mathbf{45}_H \rangle \sim \langle X \rangle \sim M_{\mathrm{GUT}}$, and $M \sim M_{\mathrm{string}}$, with X being an SO(10) singlet and $\langle \mathbf{45}_H \rangle$ being proportional to $B - L$. As mentioned before, $\langle H_V \rangle \sim 1$ TeV $> \langle H_s \rangle \sim \langle H_u \rangle \sim 200$ GeV $\gg \langle H_d \rangle$. $\mathbf{10}_H$ contains the Higgs doublets H_u and H_d of MSSM. While $H_u = \mathbf{10}^u_H$, the down type Higgs is contained partly in $\mathbf{10}^d_H$ and partly in $\mathbf{16}^d_H$ (see Ref. [10]) – that is $H_d = \cos\gamma \mathbf{10}^d_H + \sin\gamma \mathbf{16}^d_H$. If $\sin\gamma = 0$, one would have $\tan\beta = m_t/m_b$, but with $\cos\gamma \ll 1$, $\tan\beta$ can have small to intermediate values of $3 - 20$. ($\tan\beta/\cos\gamma = m_t/m_b$ is fixed.) The entries in Eq. (3) with a factor $1/M$ are suppressed by $M_{\mathrm{GUT}}/M \sim 1/10$. Note however that the contribution from h_{3V} and h'_{3V}/M terms to the down quark and charged lepton mass matrices could be comparable, $\cos\gamma \sim 1/10$, which is what we adopt.

One can verify that Eq. (3) will induce mass–matrices of the type shown in Eqs. (1) and (2) with definite correlations among X_f, X'_f, Y_c and Y'_c sectors. To see these correlations, it is useful to block–diagonalize the 5×5 mass matrix given by Eq. (2) and its analogs, so that the light families (e, μ and τ) get decoupled from the heavy ones. From now on, we denote the gauge basis (in which Eqs. (1) and (2) are written) by $\psi^0_{L,R}$ and the transformed basis which yields the block–diagonal form by $\psi'_{L,R}$. Given the SO(10) group structure of Eq. (3), it is easy to see that the effective Dirac mass matrices of the muon and the tau families in the up, down, charged lepton and neutrino sectors, resulting from block-diagonalization, would have the following form at the GUT scale [23]:

$$
\begin{aligned}
M_u &= \begin{pmatrix} 0 & -\epsilon \\ \epsilon & 1 \end{pmatrix} \mathcal{M}_u^0, \qquad M_d = \begin{pmatrix} 0 & \eta-\epsilon \\ \eta+\epsilon & 1+\xi \end{pmatrix} \mathcal{M}_d^0, \\
M_\nu^D &= \begin{pmatrix} 0 & 3\epsilon \\ -3\epsilon & 1 \end{pmatrix} \mathcal{M}_u^0, \quad M_l = \begin{pmatrix} 0 & \eta+3\epsilon \\ \eta-3\epsilon & 1+\xi \end{pmatrix} \mathcal{M}_d^0.
\end{aligned} \tag{4}
$$

Here the matrices are written in the primed basis (see above), so that the Lagrangian is given by $\mathcal{L} = \overline{\psi}'_R \mathcal{M} \psi'_L + h.c.$. (These should be compared with the transpose of the corresponding matrices in Ref. [10].) It is easy to verify that the entries $1.\mathcal{M}_u^0$, ξ, ϵ, η and $1.\mathcal{M}_d^0$ are proportional respectively to $h_{3V}h_{3\bar{V}}/h_V$, $h'_{3V}h_{3\bar{V}}/h_V$, $h_{2V}h_{3\bar{V}}/h_V$, $h'_{2V}h_{3\bar{V}}/h_V$ and $h_{3V}h_{3\bar{V}}/h_V$. (For example $(\mathcal{M}_u^0, \mathcal{M}_d^0) \simeq (m_t^0, m_b^0) \simeq (2h_{3V}h_{3\bar{V}}/h_V)(v_u, v_d)(v_s/v_0)$.) Note that the $(B-L)$-dependent antisymmetric parameter ϵ arises because $\langle \mathbf{45}_H \rangle \propto B-L$.

The eight p-parameters of Eq. (2) and its analogs can be readily obtained from Eqs. (3) and (4). They are:

$$
\begin{aligned}
p_u &= -2\epsilon,\ p'_u = 2\epsilon,\ p_d = \frac{2(\eta-\epsilon)}{1+\xi},\ p'_d = \frac{2(\eta+\epsilon)}{1+\xi}, \\
p_\nu &= 6\epsilon,\ p'_\nu = -6\epsilon,\ p_l = \frac{2(\eta+3\epsilon)}{1+\xi},\ p'_l = \frac{2(\eta-3\epsilon)}{1+\xi}\ .
\end{aligned} \tag{5}
$$

As mentioned earlier, there are only three independent parameters (η, ϵ, ξ), leading to non-trivial correlations between observables.

The matrices of Eq. (4) can be diagonalized in the approximation $\epsilon, \eta \ll \xi, 1$. One obtains

$$
\begin{aligned}
m_b^0 m_\tau^0 &\simeq 1 - 8|\epsilon|^2|1+\xi|^2, \quad m_c^0 m_t^0 \simeq |\epsilon|^2, \quad m_s^0 m_b^0 \simeq |\eta^2-\epsilon^2||1+\xi|^2, \\
m_\mu^0 m_\tau^0 &\simeq |\eta^2 - 9\epsilon^2||1+\xi|^2, \qquad\qquad |V_{cb}^0| \simeq |\epsilon\xi - \eta||1+\xi|\ .
\end{aligned} \tag{6}
$$

Here the superscript "0" denotes that these relations hold at the unification scale. A reasonably good fit to all observables can be obtained (details of this discussion will be given in a separate paper [20]) by choosing

$$
\epsilon = -0.05,\ \eta = 0.0886,\ \xi = -1.45, \tag{7}
$$

which leads to [24] $m_\mu^0/m_\tau^0 \simeq 1/17.5$, $m_s^0/m_b^0 \simeq 1/44.5$, $m_c^0/m_t^0 \simeq 1/400$, $|V_{cb}^0| \simeq 0.031$. After renormalization group extrapolation is used (using $\tan\beta = 10$ for definiteness) these values lead to $m_c(m_c) = 1.27$ GeV, $|V_{cb}| = 0.036$, $m_s(1\ \text{GeV}) = 160$ MeV, all of which are in good agreement with observations (to within 10%). Owing to the larger QCD effect in ESSM compared to MSSM, the predicted value of $m_b(m_b)$ is about 20% larger than the experimentally preferred value. Allowance for either larger values of $\tan\beta$ ($\approx 35-40$) [25], or gluino threshold corrections, and/or a 20% $B-L$ dependent correction to the vector family mass at the GUT scale (see Ref. [16]) could account for such a discrepancy.

Eqs. (5) and (7) lead to [26]:

$$
\begin{aligned}
p_\nu = -0.30, \quad p'_\nu = +0.30, \quad p_l = 0.282, \quad p'_l = -1.05 \\
p_u = 0.10, \quad p'_u = -0.10, \quad p_d = -0.607, \quad p'_d = -0.163\ .
\end{aligned} \tag{8}
$$

Now, the light neutrino masses are induced by the seesaw mechanism. In addition to Eq. (3), there are terms in the superpotential that induce heavy Majorana masses for the right handed neutrinos of the three chiral families: $W' \supset \nu_R'^T M_R^\nu \nu_R'$. Let us assume that the matrix M_R^ν for the dominant $\nu_\mu - \nu_\tau$ sector has the simple form

$$M_R^\nu = M_R \begin{pmatrix} 0 & y \\ y & 1 \end{pmatrix}$$

as in Ref. [10]. (We are ignoring here the masses and mixings of the first family. Their inclusion will modify the present discussion only slightly.) The effective light neutrino mass matrix for the $\nu_\mu - \nu_\tau$ sector is them

$$M_\nu^{\text{light}} = 1 y^2 M_R \begin{pmatrix} 0 & y\epsilon^2 \\ y\epsilon^2 & \epsilon^2 + 2\epsilon y \end{pmatrix} . \tag{9}$$

The $\nu_\mu - \nu_\tau$ oscillation angle is then

$$\theta_{\nu_\mu \nu_\tau} \simeq \left| y\epsilon^2 \epsilon^2 + 2\epsilon y - \eta - 3\epsilon 1 + \xi \right| . \tag{10}$$

The second term in Eq. (10), $(\eta - 3\epsilon)/(1+\xi) \simeq -0.53$, arises from the charged lepton sector, while the first term, arising from the neutrino sector is approximately equal to $\sqrt{m_{\nu_2}/m_{\nu_3}}$. Varying m_{ν_2}/m_{ν_3} in the range $(1/25 - 1/8)$, so as to be compatible with the solar and the atmospheric neutrino oscillation data, we find that $y = (1/42.5$ to $1/44.8)$, and for this range of y, $\sin^2 2\theta^{\text{osc}}_{\nu_\mu \nu_\tau} \approx (0.95 - 1.0)$. (Such a hierarchical value of y is nicely consistent with flavor symmetries that were assumed in the Dirac mass matrices.)

4 NuTeV Anomaly and LEP Neutrino Counting in the ESSM Framework

Having described the general framework, we now proceed to show how ESSM modifies expectations for neutral current interactions at NuTeV as well as neutrino counting at LEP. Since the system is quite constrained by its structure and symmetries, we will see that there are correlations not only between NuTeV and LEP, but also in charged current interactions. To see these, we have to go from the gauge basis in which the mass matrices of Eqs. (1) and (2) are written to the mass eigenbasis for the charged and the neutral leptons. The same transformation should then be applied to the neutral currents and the charged currents.

The diagonalization of the mass matrices can be carried out in two steps. Consider the Lagrangian term $\overline{\psi}^0_R M \psi^0_L$, where $\psi^0_{L,R}$ may stand for fermions in any of the four sectors of ESSM in the gauge basis. The mass matrix M has a form as shown in Eq. (2). Let us write it as

$$M = \begin{pmatrix} 0 & X \\ Y & Z \end{pmatrix},$$

where 0 is a 3×3 block matrix with all its entries equal to zero, X is a 3×2 matrix, Y is a 2×3 matrix and Z is a 2×2 matrix. The transformation $\psi'_{L,R} = U_{L,R}\psi^0_{L,R}$, where

$$U_{L,R} = \begin{pmatrix} 1 - 12\rho_{L,R}\rho^\dagger_{L,R} & \rho_{L,R} \\ -\rho^\dagger_{L,R} & 1 - 12\rho^\dagger_{L,R}\rho_{L,R} \end{pmatrix} + O(\rho^3_{L,R}) \tag{11}$$

and $\rho^\dagger_L = Z^{-1}Y$, $\rho^\dagger_R = XZ^{-1}$ will bring M to a block-diagonal form, in which the three light chiral families get decoupled from the two heavy ones. The effective mass matrix from the light sector is given as $M_{\text{light}} = -XZ^{-1}Y$.

Let us apply the procedure just described to the charged lepton sector. The effective μ'-τ' mixing matrix is found to be

$$M_{\mu'-\tau'} = \begin{bmatrix} 0 & p_l\kappa_d\kappa'_s/M_E \\ p'_l\kappa'_d\kappa_s/M'_E & \kappa_d\kappa'_s/M_E + \kappa'_d\kappa_s/M'_E \end{bmatrix} \tag{12}$$

where $M_E = M_N$ [see Eq. (1)] by $SU(2)_L$ symmetry. The physical μ and τ leptons, denoted by $\mu_{L,R}$ and $\tau_{L,R}$ are then

$$\begin{aligned} \mu'_{L,R} &= c_{L,R}\ \mu_{L,R} + s_{L,R}\ \tau_{L,R} \\ \tau'_{L,R} &= -s_{L,R}\ \mu_{L,R} + c_{L,R}\ \tau_{L,R} \end{aligned} \tag{13}$$

with $c_L = \cos\theta_L$, $s_L = \sin\theta_L$ etc. From Eq. (12) we have $\theta_R \simeq p_l/2$ and $\tan\theta_L \simeq p'_l/2$ (where we have set $\kappa_s = \kappa'_s$, $\kappa_d = \kappa'_d$). Note that $\mu_L - \tau_L$ mixing can be quite large in our framework [see Eq. (8)], while θ_R is small (so that the correct $\mu - \tau$ mass hierarchy is reproduced), hence the use of $\tan\theta_L$, rather than θ_L,

Applying the same transformation to the relevant neutral current of the charged lepton: "J_{Z^0}" $= \overline{\psi}^0_L diag.(1,1,1,1,a_L)\psi^0_L + \overline{\psi}^0_R diag.(1,1,1,a_R,1)\psi^0_R$, where $\psi^0_{L,R} = (e^0, \mu^0, \tau^0, E^0, E'^0)_{L,R}$, will lead to the following *new couplings* of the Z^0 boson to the leptons (i.e., in addition to their Standard Model couplings):

$$\begin{aligned} \Delta\mathcal{L}^{\text{leptons}}_{NC} = gZ^0 2\cos\theta_W(a_L - 1)\eta^2_d \big[&(c_L p'_l - s_L)^2\overline{\mu}_L\mu_L + (s_L p'_l + c_L)^2\overline{\tau}_L\tau_L \\ &+ (c_L p'_l - s_L)(s_L p'_l + c_L)(\overline{\mu}_L\tau_L + \overline{\tau}_L\mu_L)\big] . \end{aligned} \tag{14}$$

Here we have defined $\eta_d = \kappa'_d/M_E$. We shall also use related quantities $\eta'_d = \kappa_d/M_{E'}$, $\eta_u = \kappa'_u/M_N$ and $\eta'_u = \kappa_u/M_{N'}$. The new interactions of Z^0 with μ_R and τ_R can be obtained from Eq. (14) by the replacement $L \to R$, $p'_l \to p_l$, $\eta'_d \to \eta_d$. Here, $a_{L,R}$ are defined as $a_{L,R} = T_3 - Q\sin^2\theta_W$.

To obtain numerical estimates of violations flavor and of universality, we note that to a very good approximation, $\eta'_u = \eta_u, \eta'_d = \eta_d$, and $\kappa'_s = \kappa_s$ (in all four sectors). We then have $\eta'_d/\eta'_u \simeq m_b/m_t \simeq 1/60$. Since violations of universality and flavor-changing effects in the up and neutrino sectors can at most be about 1-2%, we expect $\eta_u \lesssim 1/8$-$1/10$. Such a magnitude for η_u is quite plausible [27]. η_d is then $\approx 1/500$, leading to extremely tiny effects in the charged lepton sectors. For example, the ratio $\Gamma(Z \to \mu^+\mu^-)/\Gamma(Z \to \tau^+\tau^-)$ deviates from the Standard Model value only by about 1 part in 10^5. The decay $Z^0 \to \mu^+\tau^-$ has a rate proportional to $\eta^4_d \sim 10^{-10}$.

Violations of flavor and universality, analogous to those in Eq. (14) exist in the quark sector as well. For charm and top quarks, such effects are larger, by a factor of $(m_t/m_b)^2$ in the amplitude, compared to the charged lepton sector. The $Z^0 \to c\overline{c}$ coupling deviates from the Standard Model value by an amount given by $p'_u\eta_u/2$ or $p_u\eta_u/2$ [28]. Owing to the smallness of p_u and p'_u ($= \pm 0.05$ in the example given in the previous section), the deviation of the rate for $Z^0 \to c\overline{c}$ from the Standard Model value is only about 10^{-4}.

The interesting feature having its origin in the SO(10) group theoretic structure of Eq. (3) is that while non-universality in neutral current interactions involving quarks and the charged leptons is extremely tiny, it is not so in the neutral lepton sector. This difference affects both NuTeV neutral current cross section and LEP neutrino counting. There are two reasons for the difference. First, LEP neutrino counting is sensitive to the $Z^0 \to \nu_\tau\overline{\nu}_\tau$ coupling (while $Z^0 \to t\overline{t}$ is kinematically forbidden). Second, since the $\nu_\mu - \nu_\tau$ oscillation angle is large, as required by SuperKamiokande, and also as predicted by our framework, the effective p'_l parameter is large (≈ -1.05), unlike the case for charm ($p'_u \approx 0.05$). To see the effects more concretely we need to diagonalize the neutral lepton mass matrix of Eq. (2), to which we now turn.

In addition to Eq. (2), the three ν_R^i fields have superheavy Majorana masses parametrized by a matrix M_R^ν. Once the ν_R^i are integrated out, small masses for N_L and N'_L fields will emerge. (There is no direct $\nu_L^i\nu_L^j$ mass term after seesaw diagonalization because of the structure of Eq. (2).) Let us write these effective mass terms (of order eV or less) as

$$\mathcal{L}_{\rm mass}^{\rm eff} = m_{11}N_L^0N_L^0 + m_{22}N_L'^0N_L'^0 + 2m_{12}N_L^0N_L'^0 \ . \tag{15}$$

If we denote $(M_R^\nu)^{-1}_{ij} = a_{ij}$, the mass terms are $m_{11} = \kappa_u^2(a_{33} + 2a_{23}p_\nu + a_{22}p_\nu^2)$, $m_{22} = \kappa_s^2 a_{33}$, $m_{12} = \kappa_u\kappa_s(a_{33} + p_\nu a_{23})$. The N_L^0 and $N_L'^0$ fields have Dirac masses of order few hundred GeV; they also posses non-diagonal Dirac mass mixing terms involving the light neutrinos. Upon identifying the light components, Eq. (15) will generate small Majorana masses of the standard left-handed neutrinos.

We can block diagonalize the Dirac mass matrix of the neutral leptons which is obtained from Eq. (2) after integrating out the superheavy ν_R^i fields. This can be done by applying a unitary transformation on $(\nu_\mu^0, \nu_\tau^0, N_L^0, N_L'^0)$ fields. Note that the $(N_R,\ N'_R)$ fields do not mix with the light neutrinos, since ν_R^i are superheavy. Define

$$\begin{aligned} a &= p'_\nu\eta_u,\ \ b = \eta_u,\ \ c = \kappa'_s/M_N = \eta_s, \\ N_2 &= \sqrt{1+a^2+b^2},\ \ N_3 = \sqrt{1+b^2+c^2},\ \ N_4 = \sqrt{(1+a^2)(1+b^2)+c^2}\ . \end{aligned} \tag{16}$$

The transformation $(\nu'_2, \nu'_3, \nu'_4, \nu'_5)^T = U^\nu(\nu_\mu^0, \nu_\tau^0, N, N')_L^T$, where

$$U^\nu = \begin{pmatrix} N_3N_4 & -abN_3N_4 & abcN_3N_4 & -a(1+c^2)N_3N_4 \\ 0 & 1N_3 & -cN_3 & -bN_3 \\ -abcN_2N_4 & c(1+a^2)N_2N_4 & N_2N_4 & -bcN_2N_4 \\ aN_2 & bN_2 & 0 & 1N_2 \end{pmatrix} \tag{17}$$

block diagonalizes the Dirac mass entries, so that there is no mixing between the massless states (ν'_2, ν'_3) and the massive states (ν'_4, ν'_5). From Eq. (17), one can read off the light mass eigenstate components in the original fields defined in the gauge basis. For example, $\nu_\mu^0 = (N_3/N_4)\nu_2' + ...$, $N_L = [abc/(N_3N_4)]\nu_2' - (c/N_3)\nu_3' + ...$, etc, where the dots denote the heavy components, which we drop since they are not kinematically accessible to NuTeV and LEP. Once these heavy components are dropped, the resulting states are not normalized to unity and it is this feature that is relevant to the NuTeV anomaly and LEP neutrino counting.

As a digression, we may mention that when the light neutrino mass matrix resulting from Eq. (15) is written in terms of (ν_2', ν_3') fields, it is given approximately (by setting $a_{33} = 0$, and $a_{23} \ll a_{22}$, both of which follow from the form of M_R^ν given before) by,

$$M_\nu^{\rm light} \simeq \begin{pmatrix} 0 & a_{23}p_\nu \\ a_{23}p_\nu & a_{22}p_\nu^2 + 4a_{23}p_\nu \end{pmatrix} \kappa_u^2 \kappa_s^2 M_N^2 \ . \tag{18}$$

Eq. (18) is of course completely equivalent to Eq. (9), except for having a reparametrization, and thus preserves the prediction of large ν_μ-ν_τ oscillation angle [see discussion below Eq. (9)].

Having identified the light neutrino states through Eq. (17), we can calculate the correction to neutrino counting at LEP. In the gauge basis, the Z^0 coupling is given by $[g/(2\cos\theta_W)]Z^0[\overline{\nu}_e^0\nu_e^0 + \overline{\nu}_\mu^0\nu_\mu^0 + \overline{\nu}_\tau^0\nu_\tau^0 + \overline{N}_L^0 N_L^0 + \overline{N}_R^0 N_R^0]$. The last term does not affect N_ν (the number of light neutrinos counted at LEP), since N_R is heavier than Z, and since it has no mixing with the light neutrinos. Applying the transformation of Eq. (17) we find

$$N_\nu({\rm LEP}) = 3 - 2\eta_u^2(1 + p_\nu'^2) \ . \tag{19}$$

The experimental value from LEP is $N_\nu = 2.9841 \pm 0.0083$ [29]. We see that ESSM leads to a *reduction* in N_ν, which is in agreement with the LEP data. Setting $p_\nu' = 0.3$ [see Eq. (8)], and $\eta_u = 1/10$-$1/15$, we have $N_\nu = 2.9782$, to 2.9903. The suggested two sigma deviation in N_ν measured at LEP compared to Standard Model may be taken as a hint for ν_τ-N' and ν_μ-N' mixings. It would imply a magnitude for $\eta_u \approx (1/10 - 1/15)$, which can then be used to predict deviations in the other experiments, such as NuTeV.

There are modifications in the charged current interactions as well, which is straightforward to compute:

$$\mathcal{L}_{cc} = g\sqrt{2}W^+[\overline{\nu}_e e_L + \{1 - 12\eta_u^2(c_L p_\nu' - s_L)^2\}\overline{\nu}_\mu \mu_L + \{1 - 12\eta_u^2(c_L + p_\nu' s_L)^2\}\overline{\nu}_\tau \tau_L] + h.c. \tag{20}$$

Here we *define* ν_μ as the normalized state that couples to $\mu_L^- W^+$ and similarly ν_τ as the normalized state that couples to $\tau_L^- W^+$. In terms of ν_2' and ν_3', ν_μ is given as

$$\nu_\mu = \cos\phi\ \nu_2' + \sin\phi\ \nu_3' \tag{21}$$

where $\sin\phi = -s_L(1 - b^2/2) - (s_L/2)(bs_L - ac_L)^2$. The state orthogonal to ν_μ, viz., $\hat{\nu}_\mu = -\sin\phi\ \nu_2' + \cos\phi\ \nu_3'$, is not exactly ν_τ. (Thus, ν_μ produced in π decays can produce τ leptons, but numerically this cross section is very small, being proportional to $\eta_u^4 \sim 10^{-5}$.)

In order to see how the model accounts for the NuTeV neutral current anomaly, it is useful to rewrite the $Z^0\overline{\nu}_i'\nu_j'$ interaction in terms of the current eigenstate ν_μ and the state orthogonal to it ($\hat{\nu}_\mu$). Suppressing the first family, it is given as

$$\begin{aligned}\mathcal{L}_{NC}^{Z} &= g2\cos\theta_W Z^0[\bar{\nu}_\mu\nu_\mu\{1-(ac_L-bs_L)^2\}+(\bar{\nu}_\mu\hat{\nu}_\mu+\bar{\hat{\nu}}_\mu\nu_\mu)\{(b^2-a^2)c_Ls_L-(c_L^2-s_L^2)ab\} \\ &+ \bar{\hat{\nu}}_\mu\hat{\nu}_\mu\{1-(bc_L+as_L)^2\}]\ . \qquad (22)\end{aligned}$$

Now, in our model, charged current interaction at NuTeV remains the same as in the Standard Model. This is because ν_μ beam is prepared in π^+ decays along with μ^+. The same ν_μ is detected in the charged current channel at NuTeV by detecting the μ^+ that it produces. Since by definition, the Fermi coupling G_μ is given from Eq. (20) in our model as

$$G_\mu\sqrt{2}^{\rm ESSM} \equiv g^2 8m_W^2[1-\eta_u^2 2(c_Lp_\nu'-s_L)^2], \qquad (23)$$

both production and detection via charged current at NuTeV are unchanged. On the other hand, the cross section $\sigma(\nu_\mu N\to\nu X)$ will be modified:

$$\sigma(\nu_\mu N\to\nu X)^{\rm ESSM}=\sigma^{\rm SM}(\nu_\mu N\to\nu X)[1-2\eta_u^2(c_Lp_\nu'-s_L)^2]\ . \qquad (24)$$

Notice that the neutral current cross section is *reduced* compared to the Standard Model. Using $\eta_u = 1/11.6$ to $1/13.3$ and $p_\nu' = 0.3$, $s_L/c_L = -0.526$ (see Eqs. (4)-(5)), we find the deviation $\sigma(\nu_\mu N\to\nu X)^{\rm ESSM}/\sigma^{\rm SM}(\nu_\mu N\to\nu X)\simeq 1-(0.006$ to $0.008)$. This is in good agreement with the reported NuTeV value [30, 1]. We stress the intimate quantitative link between the reduction in LEP neutrino-counting N_ν and that in the NuTeV cross section [see Eqs. (19) and (24)], which emerges because all the relevant parameters are fixed owing to our considerations of fermion masses and mixings. It is worth noting that owing to hierarchical masses of the three families, and thus nonuniversal mixings of (ν_e, ν_μ and ν_τ) with N', the reduction in N_ν is *not* simply three times the reduction in $\sigma(\nu_\mu N\to\nu X)$.

We now turn to the question of universality in charged current processes. The flavor dependence of the charged current couplings predicted by our framework, Eq. (22), will lead to nonuniversality in leptonic decays, correlated with the ν_μ-nucleon neutral current cross section measured at NuTeV, as well as neutrino counting at LEP. To estimate these effects, we recall that the Fermi coupling G_μ determined from muon decay, is to be identified with the right side of Eq. (23), in our framework. The corresponding coupling for β-decay and $\pi^+\to e^+\nu_e$ decay, G_β, is given as in the Standard Model, $G_\beta/\sqrt{2}=g^2/(8m_W^2)$, leading to the relation

$$G_\beta G_\mu = 1+\eta_u^2 2(c_Lp_\nu'-s_L)^2\ . \qquad (25)$$

This modification leads to a rescaling of the CKM matrix element $|V_{ud}|$ determined from β decay by a factor $(\eta_u^2/2)(c_Lp_\nu'-s_L)^2$. This deviation is exactly a quarter of the deviation in $\nu_\mu - N$ neutral current cross section measured at NuTeV and thus in the range 0.15-0.25%. Such a small departure in $|V_{ud}|$ is fully consistent with unitarity constraints on the 3×3 CKM matrix.

The leptonic decays of π^+ mesons provide a more sensitive probe of e-μ universality. In the Standard Model, the branching ratio $R_{e\mu}^{\rm SM} \equiv \Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu)$ has been computed quite accurately, including radiative corrections to be [31] $R_{e/\mu}^{\rm SM} = (1.2352 \pm 0.0004) \times 10^{-4}$. In our framework, this prediction is modified to

$$R_{e/\mu}^{\rm ESSM} = R_{e/\mu}^{\rm SM}[1 + \eta_u^2(c_L p'_\nu - s_L)^2] \ . \tag{26}$$

The PSI experiment [32] measures this ratio to be $R_{e/\mu}^{\rm exp-PSI} = (1.2346 \pm 0.0050) \times 10^{-4}$, whereas the TRIUMF experiment [33] finds it to be $R_{e/\mu}^{\rm exp-TRIUMF} = (1.2285 \pm 0.0056) \times 10^{-4}$. If we choose $\eta_u^2(c_L p'_\nu - s_L)^2$ =0.3 to 0.4%, so that the deviation from the Standard Model in ν_μ–nucleon neutral current scattering at NuTeV is 0.6 to 0.8%, we have $R_{e/\mu}^{\rm ESSM} = (1.2389$ to $1.2401) \times 10^{-4}$. This value is about 0.86 to 1.1 sigma above the PSI measurement, and about 1.8 to 2.1 sigma above the TRIUMF measurement. We consider these deviations, although not insignificant for the TRIUMF experiment, to be within acceptable range. We find it exciting that modest improvements in these measurements can either confirm or entirely exclude our explanation of the NuTeV anomaly.

It should also be mentioned that $e - \mu$ universality is well tested in $\tau^+ \to e^+\nu_e\bar{\nu}_\tau$ versus $\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau$ decays as well. The effective Fermi coupling strength $G_{\tau e}$ and $G_{\tau\mu}$ characterizing these decays are in the ratio [34] $G_{\tau e}/G_{\tau\mu} = 0.9989 \pm 0.0028$. Our framework predicts it to be $(G_{\tau e}/G_{\tau\mu})^{\rm ESSM} = (G_{\tau e}/G_{\tau\mu})^{\rm SM}[1 + (\eta_u^2/2)(c_L p'_\nu - s_L)^2]$. Using the correction factor in this ratio to be 0.15 to 0.2% (so that deviation at NuTeV is 0.6 to 0.8%), we find the deviation from experiment to be at the level of 0.9 to 1.1 sigma, which is quite acceptable.

5 Concluding remarks

The ESSM framework we have adopted here has been motivated on several grounds, as noted in our earlier papers [2, 3] and summarized here in the introduction. Within this framework, we have shown that the mixing of ν_μ and ν_τ with the singlet lepton N' modifies ν_μ neutral current interactions as well as LEP neutrino counting. The recently reported 3 sigma anomaly in ν_μ-nucleon scattering at NuTeV can be explained in a simple way in our framework in terms of ν_μ-N' mixing [35,36]. This explanation of the NuTeV anomaly leads to a predicted *decrease* in LEP neutrino counting, bringing the measured value [29], which is 2 sigma below the Standard Model prediction, to better agreement with theory. The ESSM framework has been embedded into an SO(10) unified theory which makes correlations among observable quantities possible. Such an embedding preserves the unification of gauge couplings and provides a quantitative understanding of the pattern of quark and lepton masses, including the smallness of V_{cb} and the largeness of the ν_μ-ν_τ oscillation angle. It is intriguing that largeness of the ν_μ-ν_τ oscillation angle makes it possible for the ESSM-framework to be relevant quantitatively to the NuTeV anomaly. It is futhermore interesting that variant patterns of SO(10)-based fermion mass-matrices, extended to the ESSM-framework, which are essentially on par with each other as regards their success in describing the masses and mixings of all fermions, can in fact be distinguished by NuTeV-type experiments [26]. In short, NuTeV can probe into GUT-scale physics. The explanation presented here for the NuTeV anomaly can be either confirmed or excluded by modest improvements in tests of $e-\mu$ universality in $\pi^{\pm}$ and $\tau^{\pm}$ decays. It can of course also be tested by improved measurements of neutrino-counting at the Z^0-peak. The hallmark of ESSM (independent of the NuTeV and LEP neutrino-counting results) is the existence of complete vectorlike families $(U, D, N, E)_{L,R}$ and $(U', D', N', E')_{L,R}$ with the masses in the range of 200 GeV to 2 TeV (say), which will certainly be tested at the LHC and a future linear collider.

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[18] They can effectivelly be admixtures of, for example, a dominant SO(10) singlet and a sub-dominant SO(10) **45**-plet with a VEV along *B-L* (see Ref. [16] for illustration).

[19] For instance going to the hat basis (see text), in which the first family is (almost) decoupled from the vectorlike families, the leading terms of the effective superpotential involving the two chiral ($\mathbf{16_2}$ and $\mathbf{16_3}$) and the two vectorlike families ($\mathbf{16_V}+\overline{\mathbf{16}}_\mathbf{V}$) that would conform with the Yukawa coupling matrix of Eq. (1) and also would yield (after Q and Q' are integrated out) an effective 3×3 mass matrix analogous to that of Ref. [10] is given by:

$$\begin{aligned}\hat{W}_{Yuk} &= h_V \mathbf{16_V}\overline{\mathbf{16}}_\mathbf{V}\mathbf{1_V} + f_V \mathbf{16_V}\overline{\mathbf{16}}_\mathbf{V}(\mathbf{45_H}/M)\mathbf{1'_V} + h_{3V}\mathbf{16_3 16_V 10_H} \\ &+ \tilde{h}_{3V}\mathbf{16_3 16_V 10_H 45_H}/M + h_{3\bar{V}}\mathbf{16_3}\overline{\mathbf{16}}_\mathbf{V} H_S + h_{2V}\mathbf{16_2 16_V 10_H}\left\langle\frac{X}{M}\right\rangle \\ &+ a_{2V}\mathbf{16_2 16_V 10_H 45_H}/M + g_{2V}\mathbf{16_2 16_V 16_H^d 16_H}/M\end{aligned}$$

(See Ref. [16] for more details.)

[20] K.S. Babu and J.C. Pati, to appear.

[21] The right-handed neutrinos of three chiral families (ν_R^e, ν_R^μ and ν_R^τ) acquire superheavy Majorana masses, which, following the familiar see-saw mechanism, leave the left-handed neutrinos light. Since RH neutrinos are superheavy, the mixing of LH neutrinos $\nu_L^{e,\mu,\tau}$ with $(N, N')_L$, obtained from Eq. (2), would not be affected by their presence.

[22] For example, if one drops the (B-L)-dependent contributions – that is with $(h_V \mathbf{16_V \overline{16}_V 1_V} + h_{3V}\mathbf{16_3 16_V 10_H} + h_{3\bar{V}}\mathbf{16_3 16_V} H_S)$ being the leading terms of the effective superpotential (see Ref. [19]) – one would get the following relations at the GUT-scale: $x_u = x_d = x_l = x_\nu = x'_u = x'_d = x'_l = x'_\nu$; $y_q = y_l = y'_q = y'_l$, and $z_f = z'_c$. In this case, the entries denoted by "1" in Eq. (2) and its analogs in the quark and charged lepton sectors will be given by just three parameters (κ_u, κ_d and κ_s) – rather than sixteen – at the GUT scale. Similar economy arises for the p parameters (see text).

[23] The electron family is essentially massless, barring small entries ($\leq$ a few MeV) in the direct $\mathbf{O}_{3\times 3}$ subsector of Eqs. (1) and (2).

[24] Since corrections to see-saw diagonalization for the $\mu - \tau$ sector are not small, especially for m_μ^0/m_τ^0, the results quoted are based on a more accurate numerical diagonalization.

[25] See for e.g., J. Bagger, K. Matchev, D. Pierce and R. Zhang, Phys. Rev. Lett. **78**, 1002 (1997).

[26] The pattern of fermion mass matrices of Ref. [10] can also be obtained within ESSM (see [16]), in which case one would obtain: $p_\nu = 0.35$, $p'_\nu = -0.80$, $p_l = 0.27$ and $p'_l = -0.87$. Although these are roughly the same order as in Eq. (8), because of differences in some crucial signs, the pattern presented in [10] does not explain the NuTeV anomaly in conjuntion with the LEP neutrino-counting, while that of Eq. (8) does. This is the main reason for considering the variant model described here. One may turn this observation around and argue that, in the context of the ESSM framework, experiments such as NuTeV can probe GUT scale physics and help distinguish between variant mechanisms for fermion mass-generation.

[27] These may be estimated by making some plausible assumptions about the Yukawa couplings as follows. Assume that, in accord with generalized flavor symmetries, the Yukawa couplings of the vectorlike and the third families to each other involving the SO(10)-singlet Higgs fields $\mathbf{1_V}$ and $\mathbf{1'_V}$ (that is h_V and $h_{3\bar{V}}$, see Ref. [22] for definitions) are "maximal' ($\sim$ 1 to 2) at the GUT-scale; while the coupling involving the $\mathbf{10_H}$ that mixes the vectorlike with the third family (i.e., the coupling h_{3V}) may be smaller than h_V by a factor of 1/3 to 1/10 (say) at the GUT-scale. As discussed in Ref. [3], this would lead to fixed point values of the Yukawa couplings generated by h_V and $h_{3\bar{V}}$ at the electroweak scale, which are given by $z_l = 0.273$, $z'_l = 0.185$, $z'_\nu = 0.152$, $y'_l = 0.251$, $y_l = 0.184$ and $y_\nu = 0.20$ (see Eq. (10) of Ref. [3]). For the choice indicated above, the couplings generated by h_{3V} – that is $x_{l,\nu}$ and $x'_{l,\nu}$ – may, however, be smaller by factors of 1/2 to 1/4 (say) than the corresponding fixed point value of ≈ 0.3 at the electroweak scale. We thus take $x_{l,\nu} \sim x'_{l,\nu} \approx (0.3)\xi^x_\nu$, where $\xi^x_\nu \approx 1$ to 1/4. These yield:

$v_0 \equiv \langle H_V \rangle = M_{E'}/z'_l \approx M_{E'}/0.185 \approx (1.6\ TeV)\rho_{E'}$, where $\rho_{E'} \equiv M_{E'}/(300\ GeV)$. Thus, $v_u/v_0 \approx 175\ GeV/v_0 \approx (1/9)(1/\rho_{E'})$. We also get: $\eta_u \equiv \kappa'_\nu/M_{N'} \approx x'_\nu v_u/M_{N'} \approx (0.3\,\xi^x_\nu)(175\ GeV)/(300\ GeV)\rho_{E'} \approx (1/5.7$ to $1/20)(1/\rho_{E'})$ for $\xi^x_\nu \approx 1$ to $1/4$.

[28] Note, despite QCD effects, the parameters η_u for the neutrinos and for up-quarks are essentially the same.

[29] LEP Electroweak Working Group Report, hep-ex/0112021.

[30] K. McFarland, NuTeV results presented at Fermilab seminar, October 26, 2001.

[31] W. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993).

[32] C. Czapek et. al., Phys. Rev. Lett. **70**, 17 (1993).

[33] D.I. Britton et. al., Phys. Rev. Lett. **68**, 3000 (1992).

[34] W. Marciano, Phys. Rev. **D60**, 093006 (1999).

[35] Alternative explanations of the NuTeV-anomaly, based for example on Z-Z' mixing have been proposed (see e.g. E. Ma and D. P. Roy hep-ph/0111385). In such an explanation, the correlation between the NuTeV-anomaly and the LEP neutrino-counting is, however, not obvious. For an interpretation based on $\nu_e - \nu_s$ oscillations with a large neutrino mass, see C. Giunti, and M. Laveder, hep-ph/0202152.

[36] After the completion of our work, which was presented by JCP at the WHEPP-7 Conference held at Allahabad, India (January 7, 2002), we came accross a paper by S. Davidson, S. Forte, P. Gambino, N. Rius and A. Strumia, [hep-ph/0112302] which also notes the correlation between the NuTeV-anomaly and charged current universality in a general context, involving heavy fermions.